

# HEAT TRANSFER FROM A ROTATING DISK TO FLUIDS OF ANY PRANDTL NUMBER

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## INTRODUCTION

The laminar flow about a rotating disk situated in a large body of quiescent fluid was first analyzed by von Karman [1]. The heat transfer from such a rotating disk has been studied by Millsaps and Pohlhausen [2] for fluids which have Prandtl numbers in the range  $0.5 < (c_v/c_p) Pr < 10$ . For gases ( $Pr = 0.72$ ), the effects of compressibility were examined by Ostrach and Thornton [3]. In the present investigation, the restriction on Prandtl number is lifted, and heat transfer results are obtained for fluids of all Prandtl numbers.

## ANALYSES

The heat transfer process in the fluid is governed by the conservation of energy principle, which takes the following form in cylindrical coordinates

$$\rho c_p \left( v_r \frac{\partial T}{\partial r} + \frac{v_\phi}{r} \frac{\partial T}{\partial \phi} + v_z \frac{\partial T}{\partial z} \right) = k \nabla^2 T \quad (1)$$

where  $T$  is the static temperature and  $\nabla^2$  is the Laplace operator. The velocities may be rephrased in terms of von Karman's similarity variables as follows:

$$\eta = \left( \frac{\omega}{\nu} \right)^{1/2} z, \quad v_r = r\omega F(\eta), \quad v_\phi = r\omega G(\eta), \quad v_z = (\omega\nu)^{1/2} H(\eta) \quad (2a)$$

and in addition, a dimensionless temperature may be defined as

$$\theta(\eta) = (T - T_\infty)/(T_w - T_\infty) \quad (2b)$$

where  $T_w$  and  $T_\infty$  respectively represent the surface temperature (a constant) and the ambient temperature. Introducing these new variables into the energy equation, there is obtained

$$TMX\#36248 \theta'' = (Pr)H\theta' \quad (3)$$

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The primes denote differentiation with respect to  $\eta$  and  $Pr$  is the Prandtl number of the fluid. From the definition of  $\theta$  as given by equation (2b), it is clear that the boundary conditions are

$$\theta(0) = 1, \quad \theta \rightarrow 0 \text{ as } \eta \rightarrow \infty \quad (4)$$

Numerical solutions of equation (3) subject to the boundary conditions (4) have been carried out on an IBM 653 electronic computer for Prandtl numbers of 0.01, 0.1, 1, 10, and 100\*. The temperature distributions corresponding to these solutions are presented in figures 1 and 2 respectively for the low and high Prandtl number ranges. Also shown on the graphs are the values of the velocity function  $H$  used as input data in the solution of equation (3). Inspection of these figures reveals that for low Prandtl numbers, the thermal boundary layer is much thicker than the velocity boundary layer. The opposite characteristic is displayed for high Prandtl numbers. This suggests a procedure for obtaining asymptotic solutions.

First, as the Prandtl number becomes very small, figure 1 indicates that  $H$  is essentially constant throughout the thermal boundary layer. Under these circumstances, the solution of equation (3) is

$$\theta = e^{Pr H(\infty) \eta} \quad (5)$$

where  $H(\infty) = -0.88447$ . In particular, the temperature derivative at the wall, needed in computing the heat transfer, is

$$(d\theta/d\eta)_{\eta=0} = -0.88447 Pr \quad (5a)$$

From figure 2, it is seen that as the Prandtl number approaches very large values, the thermal boundary layer is confined to a smaller and smaller portion of the velocity boundary layer. This prompts us to write  $H$  in terms of a

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\*The values of  $H(\eta)$  needed to carry out the solutions were obtained by re-solving von Karman's differential equations.

series expansion about  $\eta = 0$ . So,

$$H = H(0) + H'(0)\eta + \frac{H''(0)}{2} \eta^2 + \dots$$

Since  $H(0) = H'(0) = 0$ , we write

$$H = H''(0) \frac{\eta^2}{2}$$

for small values of  $\eta$ . A solution of equation (3) corresponding to this approximation for  $H$  is

$$\theta = 1 - \frac{\int_0^{[-Pr H''(0)/6]^{1/3} \eta} \exp(-\xi^3) d\xi}{\int_0^\infty \exp(-\xi^3) d\xi} \quad (6)$$

The dimensionless temperature derivative at the wall associated with this solution is

$$(d\theta/d\eta)_{\eta=0} = [-Pr H''(0)/6]^{1/3} / \Gamma(4/3) = 0.62048 Pr^{1/3} \quad (6a)$$

where the value -1.02046 has been used for  $H''(0)$ , and  $\Gamma$  represents the gamma function as found in numerous mathematical tables.

#### HEAT TRANSFER RESULTS

The local rate of heat transfer from the disk to the fluid may be evaluated using Fourier's Law, i.e.,

$$q = -k(\partial T / \partial z)_{z=0} \quad (7)$$

Introducing a local heat transfer coefficient  $h$  by the definition

$$h = q / (T_w - T_\infty)$$

and evaluating equation (7) in terms of the variables of the analysis, there is obtained

$$\frac{h\left(\frac{\nu}{\omega}\right)^{1/2}}{k} = - \left(\frac{d\theta}{d\eta}\right)_{\eta=0} \quad (7a)$$

Since  $(d\theta/d\eta)_{\eta=0}$  depends only on the Prandtl number, it is seen that for a given fluid, the variation of the heat transfer coefficient with angular velocity is given by

$$h \sim \omega^{1/2}$$

Further,  $h$  is constant over the disk surface.

Using the numerical solutions of equation (3), the dimensionless heat transfer results are listed in table I.

TABLE I  
Dimensionless Heat Transfer Results

| Pr   | $h\left(\frac{\nu}{\omega}\right)^{1/2}/k$ |
|------|--|
| 0.01 | 0.0087051                                  |
| 0.1  | 0.076581                                   |
| 1    | 0.39625                                    |
| 10   | 1.1341                                     |
| 100  | 2.6871                                     |

For the limiting situations of very low and of very high Prandtl numbers, the following asymptotic expressions for the heat transfer may be written utilizing equations (5a) and (6a)

$$h\left(\frac{\nu}{\omega}\right)^{1/2}/k = 0.88447 \text{ Pr}, \quad \text{Pr} \rightarrow 0 \quad (8a)$$

$$h\left(\frac{\nu}{\omega}\right)^{1/2}/k = 0.62048 \text{ Pr}^{1/3}, \quad \text{Pr} \rightarrow \infty \quad (8b)$$

A graphical presentation of the heat transfer results is given on figure 3. The results have been plotted in two different ways, one appropriate to low Prandtl number fluids and the other appropriate to high Prandtl number fluids. The asymptotic lines are also shown (dashed). It is seen that the low Prandtl number asymptote already closely coincides with the computed curve at  $Pr = 0.01$ , the deviation being only 1.5%. The high Prandtl number heat transfer results approach their asymptote somewhat less rapidly, the difference between the computed and asymptotic curves being about 6.5% at  $Pr = 100$ ; this deviation is not large from the practical point of view. The significant fact is that the asymptotic curves provide sufficiently accurate heat transfer results outside the Prandtl number range for which the numerical solutions have been obtained.

By utilizing either the numerically-computed results or the asymptotic expressions, heat transfer calculations can be carried out for fluids of any Prandtl number.

#### REFERENCES

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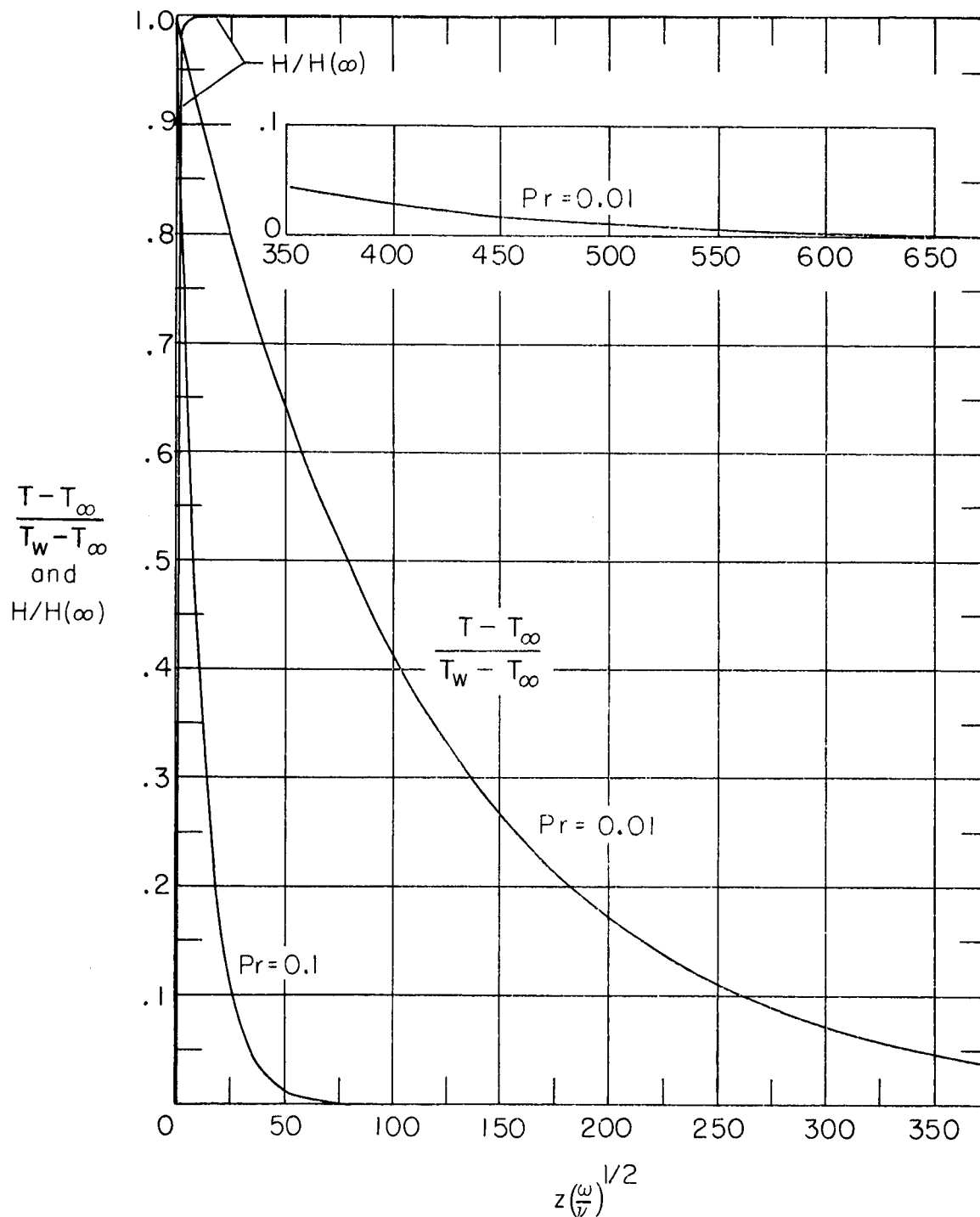


Fig. 1. - Temperature distributions for low Prandtl number fluids  
 ( $H/H(\infty)$  is input function for temperature solutions).

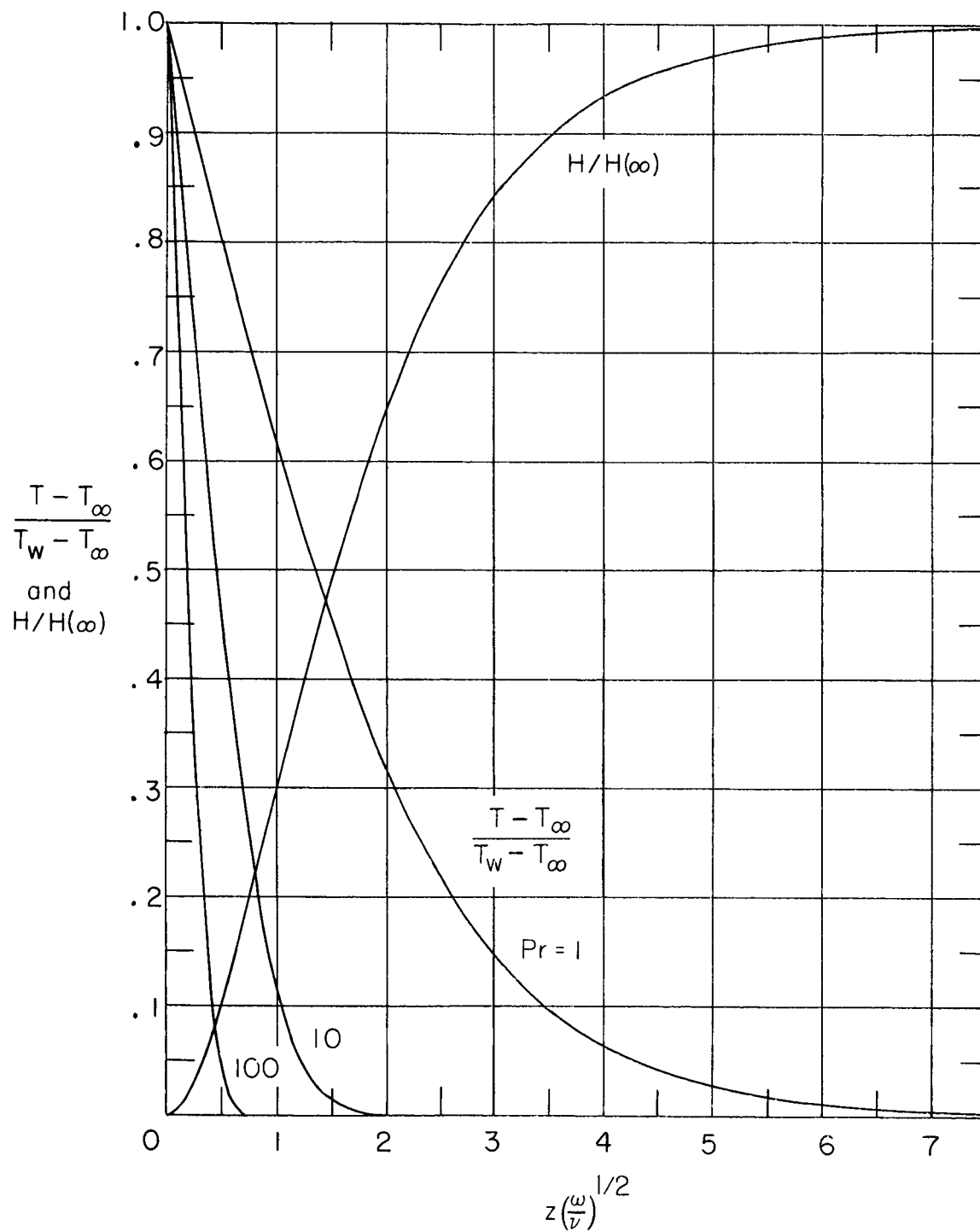


Fig. 2. - Temperature distributions for high Prandtl number fluids  
( $H/H(\infty)$  is input function for temperature solutions).

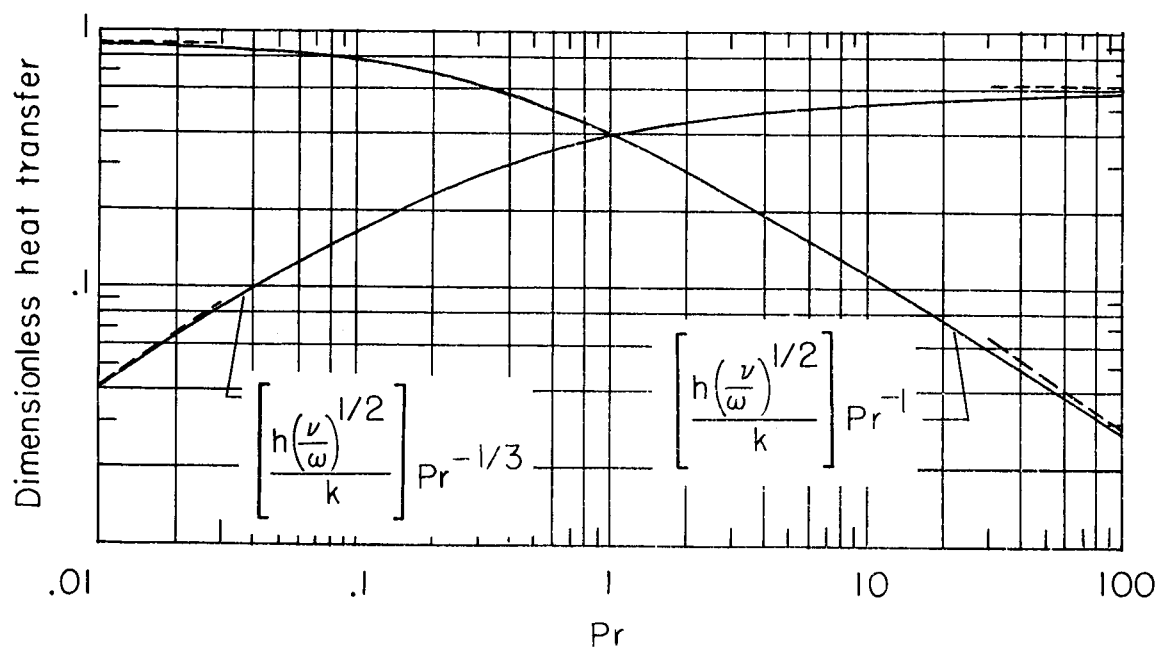


Fig. 3. - Heat transfer results.